BRIEF SPACE NOTE

Euclid conceived the space of reality as geometrical. This space is constituted of infinite equidistant points related to each other in a manner of continuous connection.

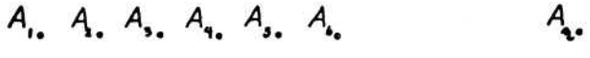
Poincaré defined this space as follows:

First, what are strictly speaking the properties of space? I mean the space that is the object of geometry and which I call geometric space. Here are some of the more essential:

1. *It is continuous;*
2. *It is infinite;*
3. *It has three dimensions;*
4. *It is homogenous, that is to say that all the points are identical to one another.*
5. *It is isotropic, that is to say that all straight lines passing through a point are identical to one another. (1)*

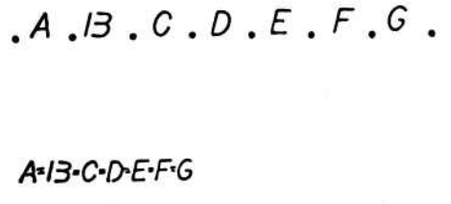
The examples that follow are very simple and somehow redundant. The first one (Fig. 1) indicates the equality of measurement or the equality of distance between the points.

FIG. 1



The second example (Fig. 2) refers to the equality of intervals between the points.

FIG. 2



The third example deals with the notion of the neighborhood of the points (The inmediate adjacent of A 1 is A 2 and between A1 and A3 only A 2 exists) This property is called continuity. (2)

Space is homogeneous because all its elements are of the same nature and is isotropic because it maintains the same properties in any direction. It is infinite because it doesn't have boundaries and it is three-dimensional because it is a composite of three variables: length, width and height. These are the most essential properties of Euclidean space.

FIG. 3

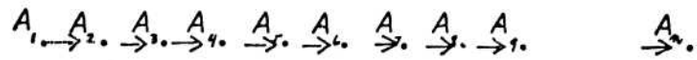


Figure 4 represents a group of elements organized in accordance with strictly Euclidean terms. This structure was familiar to artists since the XV century. Leonardo in *Il trattato della Pittura* advised the following:

If you wish to learn correct and good positions for your figures, make a frame that is divided into squares by threads and put it between your eye and the nude you are drawing, and you will trace the same squares lightly onto your paper on which you intend to draw your nude. Then place a small wax pellet on some spot on the net to serve as a marker which each time you look at the nude, you will place at the hollow of the throat (or, if he is seen from the back, over one of the vertebrae of the neck); and these threads will tell you for each position of the body, which parts of the body are precisely below the hollow of the throat . . .

It is obvious that the use of the grid for regulating the space of the rectangular canvas has been common to the artist for some time.

FIG.4

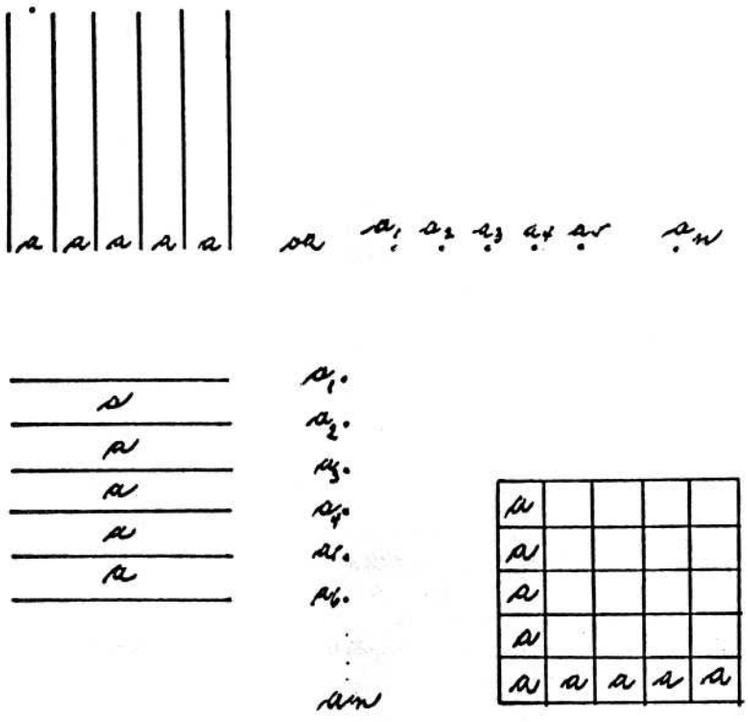


FIG. 5.

In the XVI century some artists tried to dislocate, so to speak, the structure of the pictorial space. The results of these *fancies* were called Anamorphosis (fig. 5). These representations, somehow, disregarded the notion of equidistance or the metrical constant of space. (The figures were disproportionate.) The distortion of the image conveyed a certain displacement of matter, and in a curious manner made the painting look more energetic than substantial. As far as I know these experiments have not been considered more than mere extravaganzas and have not been given much attention. (3)

Consequently, the arrangement of elements in accordance with the Euclidean scheme seems to have prevailed in most explorations in the rectangular canvas.

In 1876, William Kingdom Clifford published a paper in which he expressed the following:

(1) That small portions of space are in fact of a nature analogous to little hills on a surface which is on the average flat; namely that the ordinary laws of geometry are not valid in them.

(2) That this property of being curved or distorted is continually being passed on from one portion of space to another after the manner of a wave.

(3) That these variations of the curvature of space is what really happens in that phenomenon which we call the motion of matter, whether ponderable or ethereal.

(4) That in the physical world nothing else takes place but this variation, subject (possibly) to the law of continuity. (4)

For Clifford, somehow, substance is an accident of space. This is the apotheosis of many centuries of thinking. In Aristotle space is identified with place and defined as adjacent boundary of the containing body. Aristotle conceived space as a contingency of matter. Clifford reversed this concept and in a very beautiful manner ended with the reign of the Euclidean conception. Space is not homogenous; it has turbulances where the laws of Euclidean geometry are not the case. The notion of regularity, then, is lost. Space behaves in a manner comparable to jelly. In Figures 6 and 7, I try to convey this notion. Since these notes are somehow paradigmatic I started the example with an image enclustered in a regular grid (Euclidean) and then transforned it, through a conformal transformation, into a *new grid*. This grid is not metrical, the intervals between its elements are not regular, the straight angles have been lost, the only possible invariant kept is that of the neighborhood of the points. This image, then, expands and contracts accordingly. In the *old order* the displacement of a body in space did not affect its metrical properties; in this new grid the image changes size continuously.

What I have described so far are conceptions of space. We shouldn't forget, nevertheless, that these conceptions are human conceptions and that space, as a form of the natural phenomenon, is independent of human thought. Whatever I have described then, is a function of our conceptual scheme. Nature is independent of human thought.

The same certainty that characterizes the relativity of motion accompanies the principle of the relativity of magnitude. We must not let our courage fail in maintaining this principle, according to which the size of a body at one moment does not determine its size at another (5)

As Sartorious von Walterhausen reports, Gauss considered the three-dimensionality of space not as an inherent quality of space, but as a specific peculiarity of the human soul. (6)

Last hint:

*The concept of space can be summarized in a telegraphic manner: Pertaining to the cosmos: Space-void-emptiness-receptivity-**Ch'i (Spirit)-heaven-Tao;** Pertaining to man: Space-emptiness-receptivity-purity-chi (Spirit)-harmony-with-the-Tao. (7)*

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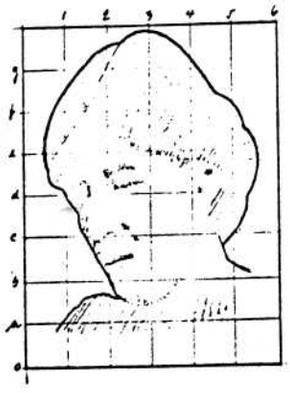
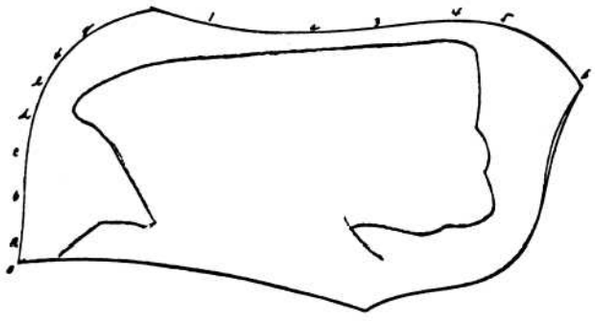


FIG. 6.



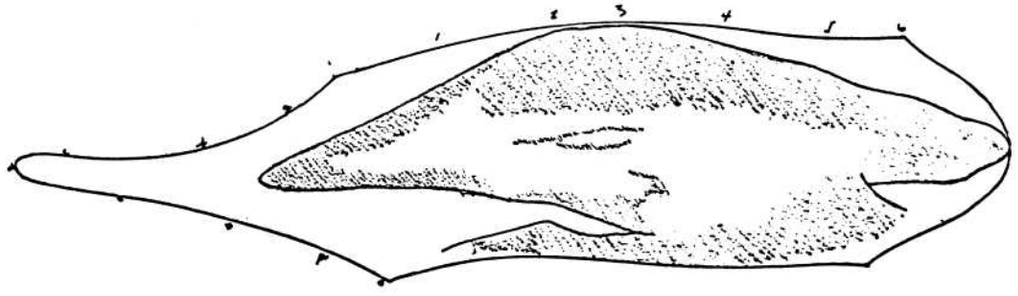


FIG.7.

Notes

(1) Henry Poincaré, *Geometry and Space.*

(2) "For this reason we endeavor to track back all conceptions of continuity to the conception 'between', i.e. to the relation A is a point of the straight line BC and lies between B and C." See Fig. 3 bottom. Hermann Weyl, *Space-Time-Matter,* p. 12.

(3) For more on Anamorphosis see Jurgis Baltrusaitis, *Anamorphoses, ou magie artificielle des effets merveilleux.* Olivier Perrin, 1969.

(4) Proceedings of the Cambridge Philosophical Society, 1876.

(5) H. Weyl, *Space-Time-Matter,* p. 283.

(6) Sartorious von Walterhausen, *Gauss zum Gedenken,* vol. 8, p. 268.

(7) Mai-Mai Sze, *The Tao of Painting,* 1679-1701, p. 95.